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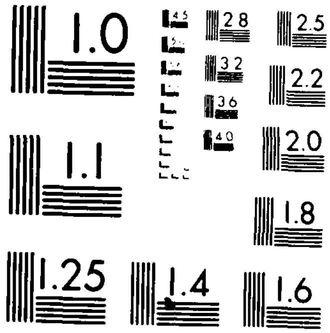

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A TWO-EQUATION TURBULENCE MODEL FOR A DISPERSED TWO-PHASED FLOW WITH VARIABLE DENSITY FLUID AND CONSTANT DENSITY PARTICLES

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Technical Report

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A two-equation turbulence model has been developed for predicting a dispersed two-phase flow with variable density fluid and constant density particles. The two equations describe the conservation of turbulence kinetic energy and its dissipation rate for the fluid. They have been derived rigorously from the momentum equations of the carrier fluid in the two-phase flow. Closure of the time-mean equations is achieved by modeling the turbulent correlations up to third order. The new model eliminates the need to simulate in an ad hoc manner the effects of the dispersed phase on turbulent structure in situations where the compressibility of the fluid must be taken into account.

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18. SUBJECT TERMS (continued)

Nonsteady Flow
Compressible Viscous Gas
Dilute Suspensions
Continuity Equation
Momentum Equation
Reynolds Time-averaging
Kinetic Energy
Dissipation Rate

PREFACE

This work was performed for the Defense Nuclear Agency under contract DNA 001-85-C-0022. Dr. George W. Ullrich was the DNA Contract Technical Manager. Dr. Allen L. Kuhl was the RDA program manager. The authors wish to thank Dr. Ullrich for his encouragement, Dr. Kuhl for suggestion of the research subject and helpful discussions, and Mrs. Vivian Fox for expert typing of the manuscript.

A-1

CONVERSION TABLE

Conversion factors for U.S. Customary to metric (SI) units of measurement

MULTIPLY → BY → TO GET
TO GET ← BY ← DIVIDE

angstrom	1.000 000 X E -10	meters (m)
atmosphere (normal)	1.013 25 X E +2	kilo pascal (kPa)
bar	1.000 000 X E +2	kilo pascal (kPa)
barn	1.000 000 X E -28	meter^2 (m^2)
British thermal unit (thermochemical)	1.054 350 X E +3	joule (J)
calorie (thermochemical)	4.184 000	joule (J)
cal (thermochemical)/cm^2	4.184 000 X E -2	mega joule/m^2 (MJ/m^2)
curie	3.700 000 X E +1	*giga becquerel (GBq)
degree (angle)	1.745 329 X E -2	radian (rad)
degree Fahrenheit	$t_K = (t°f + 459.67)/1.8$	degree kelvin (K)
electron volt	1.602 19 X E -19	joule (J)
erg	1.000 000 X E -7	joule (J)
erg/second	1.000 000 X E -7	watt (W)
foot	3.048 000 X E -1	meter (m)
foot-pound-force	1.355 818	joule (J)
gallon (U.S. liquid)	3.785 412 X E -3	meter^3 (m^3)
inch	2.540 000 X E -2	meter (m)
jerk	1.000 000 X E +9	joule (J)
joule/kilogram (J/kg) (radiation dose absorbed)	1.000 000	Gray (Gy)
kilotons	4.183	terajoules
kip (1000 lbf)	4.448 222 X E +3	newton (N)
kip/inch^2 (ksi)	6.894 757 X E +3	kilo pascal (kPa)
ktap	1.000 000 X E +2	newton-second/m^2 (N-s/m^2)
micron	1.000 000 X E -6	meter (m)
mil	2.540 000 X E -5	meter (m)
mile (international)	1.609 344 X E +3	meter (m)
ounce	2.834 952 X E -2	kilogram (kg)
pound-force (lbs avoirdupois)	4.448 222	newton (N)
pound-force inch	1.129 848 X E -1	newton-meter (N·m)
pound-force/inch	1.751 268 X E +2	newton/meter (N/m)
pound-force/foot^2	4.788 026 X E -2	kilo pascal (kPa)
pound-force/inch^2 (psi)	6.894 757	kilo pascal (kPa)
pound-mass (lbm avoirdupois)	4.535 924 X E -1	kilogram (kg)
pound-mass-foot^2 (moment of inertia)	4.214 011 X E -2	kilogram-meter^2 (kg·m^2)
pound-mass/foot^3	1.601 846 X E +1	kilogram/meter^3 (kg/m^3)
rad (radiation dose absorbed)	1.000 000 X E -2	**Gray (Gy)
roentgen	2.579 760 X E -4	coulomb/kilogram (C/kg)
shake	1.000 000 X E -8	second (s)
slug	1.459 390 X E +1	kilogram (kg)
torr (mm Hg, 0° C)	1.333 22 X E -1	kilo pascal (kPa)

*The becquerel (Bq) is the SI unit of radioactivity; 1 Bq = 1 event/s.
**The Gray (Gy) is the SI unit of absorbed radiation.

TABLE OF CONTENTS

TABLE OF CONTENTS (CONCLUDED)

Appendices

SECTION 1

INTRODUCTION

There are many problems of practical interest which require a mathematical model for a two-phase turbulent flow of suspended particles in a viscous fluid. Most one- and two-equation models for two-phase flows (e.g., Refs. 1 and 2) are based on ad hoc modifications of the single-phase turbulence-kinetic-energy and length-scale equations and fail to adequately predict the physical behavior of two-phase flows. Recently, Elghobashi et al. (Ref. 3) presented a rigorous derivation of the turbulence-energy and dissipation-rate equations from the momentum equations for an incompressible dispersed two-phase flow and successfully predicted the main features of a round gaseous jet laden with uniform-size solid particles (Refs. 4 and 5). In situations such as the dust transport by turbulence in nuclear burst flow fields (Ref. 6) and supersonic nozzle flows (Ref. 7) the compressibility of the fluid must be taken into account. The objective of the present work is to extend the two-equation turbulence model in Ref. 3 to become applicable to a compressible dispersed two-phase flow in which the fluid has variable density and the particles have constant density. The extended model can also be used to study such problems as compressible adiabatic mixing and low-speed isothermal mixing of two dissimilar two-phase flows (Ref. 8).

SECTION 2

EQUATIONS OF MOTION

We begin the formulation of the problem by stating the assumptions involved in deriving the equations. These are:

1. Both phases behave macroscopically as a continuum, but only the carrier fluid behaves microscopically as a continuum with variable density. This means that the volume-averaged equations are based on a control volume larger than the particle spacing but much smaller than the characteristic volume of the flow system. Mutual exclusion of the phases is also ensured.

2. The dispersed phase consists of rigid particles spherical in shape, uniform in size and constant in density. The uniformity of size reduces the magnitude of bookkeeping at this stage of the work, and thus concentrates the effort on understanding the mechanisms of interactions between the two phases. Extension to nonuniform size distribution is a straightforward matter (Ref 9).

3. The volume fraction of the dispersed phase is such that no collisions occur between the particles. This assumption renders the equations valid only for dilute suspensions.

4. Neither the suspended matter nor the carrier fluid undergoes any phase changes. Although this assumption rules out some situations of practical

interest, it is necessary to investigate complexities in a stepwise manner.

5. Additional assumptions on modeling of some of the turbulent correlations are stated in Sections 4, 5 and 6. The sparseness of experimental data for variable density flows allows us to assume that the forms and values of the coefficients of the turbulent correlations of constant-density flows apply to variable-density flows as well.

The instantaneous, volume-averaged momentum equations, in Cartesian tensor notations, of the carrier fluid are thus (Refs. 3 and 10)

$$(Q_1 U_i)_{,t} + (Q_1 U_j U_i)_{,j} = -(1-K\Phi_2)P_{,i} - KF\Phi_2(U_i - V_i)$$
$$+ \left[\mu_1 \Phi_1 (U_{i,j} + U_{j,i})\right]_{,j}$$
$$- \tfrac{2}{3}(\mu_1 \Phi_1 U_{\ell,\ell})_{,i} + Q_1 g_i + F_{1i} \quad . \qquad (1)$$

The corresponding equations for the particle phase are

$$(Q_2 V_i)_{,t} + (Q_2 V_j V_i)_{,j} = -\Phi_2 P_{,i} + F\Phi_2(U_i - V_i)$$
$$+ \left[\mu_2 \Phi_2 (V_{i,j} + V_{j,i})\right]_{,j} - \tfrac{2}{3}(\mu_2 \Phi_2 V_{\ell,\ell})_{,i}$$
$$+ Q_2 g_i + F_{2i} \quad . \qquad (2)$$

The continuity equations are

$$Q_{1,t} + (Q_1 U_i)_{,i} = 0 \qquad (3)$$

for the fluid and

$$Q_{2,t} + (Q_2 V_i)_{,i} = 0 \qquad (4)$$

for the particle phase. The global continuity equation is

$$\Phi_1 + \Phi_2 = 1 = \frac{Q_1}{\rho_1} + \frac{Q_2}{\rho_2} \qquad (5)$$

In Eqs. (1) - (5) and throughout the report, the subscripts 1 and 2 denote the fluid and particle phase respectively. Partial derivatives are represented by a subscript consisting of a comma and an index [e.g., $(\ )_{,t} \equiv \partial(\ )/\partial t$; $U_{i,j} \equiv \partial U_i/\partial x_j$; $U_{i,k\ell} = \partial^2 U_i/\partial x_k \partial x_\ell$ where x_i $(i = 1,2,3)$ are the rectangular spatial coordinates]. U_i are the velocity components of the fluid. V_i are the velocity components of the particle phase. ρ and μ are the material density and viscosity. $Q_i = \rho\Phi_i$ is the apparent density. Φ is the volume fraction. P is the pressure. g_i is the component of gravitational acceleration in the i direction. F_i is the component of body forces other than that due to gravity, and F is the interphase friction coefficient ($=18\mu_1/d^2$ for Stokes' flow around a particle of diameter d). K is the local effectiveness of momentum transfer from the particle phase to the fluid and is discussed in detail in Ref. 10. It suffices here to state that K equals unity for accelerated particle phase and assumes lower values when this phase is decelerated; the minimum value of K being zero. In general, K depends on the local properties of the fluid and

turbulence, the slip velocity, and the particulate size and concentration.

The mean flow equations are now obtained from the instantaneous ones, for variable ρ_1, constant ρ_2 and μ_1 and zero μ_2 (consistent with the dilute suspension approximation stated in Ref. 10, p. 256), by performing the conventional Reynolds time-averaging of Eqs. (1) to (5). (The density-weighted averaging of Favre, Ref. 11, does not render significant simplification. We use Reynolds averaging mainly because most experimental data refer to time averaging correlations). The mean momentum equations of the fluid are

$$
\begin{aligned}
(Q_1 U_i + \overline{q_1 u_i})_{,t} + (Q_1 U_j U_i)_{,j} = &-(1-K\Phi_2)P_{,i} + K\overline{\phi_2 p_{,i}} \\
&- KF\left[\Phi_2(U_i - V_i) + \overline{\phi_2(u_i - v_i)}\right] \\
&+ \mu_1\left[\Phi_1(U_{i,j}+U_{j,i}) + \overline{\phi_1(u_{i,j}+u_{j,i})}\right]_{,j} \\
&- \frac{2}{3}\mu_1(\Phi_1 U_{\ell,\ell} + \overline{\phi_1 u_{\ell,\ell}})_{,i} \\
&- (Q_1\overline{u_i u_j} + U_i\,\overline{q_1 u_j} + U_j\,\overline{q_1 u_i} + \overline{q_1 u_i u_j})_{,j} \\
&+ Q_1 g_i + F_{1i} \quad . \qquad (6)
\end{aligned}
$$

The mean momentum equations of the particle phase are

$$(Q_2V_i+\overline{q_2v_i})_{,t} + (Q_2V_jV_i)_{,j} = -[\Phi_2P_{,i}+\overline{\phi_2p_{,i}}]$$

$$+ F[\Phi_2(U_i-V_i) + \overline{\phi_2(u_i-v_i)}]$$

$$- (Q_2\overline{v_iv_j} + V_i\overline{q_2v_j} + V_j\overline{q_2v_i} + \overline{q_2v_iv_j})_{,j}$$

$$+ Q_2g_i + F_{2i} \quad . \tag{7}$$

The mean continuity equation of the fluid is

$$Q_{1,t} + (Q_1U_i + \overline{q_1u_i})_{,i} = 0 \quad . \tag{8}$$

The mean continuity equation of the particle phase is

$$Q_{2,t} + (Q_2V_i + \overline{q_2v_i})_{,i} = 0 \tag{9}$$

which can be written as

$$\Phi_{2,t} + (\Phi_2V_i + \overline{\phi_2v_i})_{,i} = 0 \tag{10}$$

since ρ_2 is constant. The mean global continuity equation is

$$\Phi_1 + \Phi_2 = 1 \tag{11}$$

which, when substrated from Eq. (5), gives

$$\phi_1 + \phi_2 = 0 \quad . \tag{12}$$

In Eqs. (6)-(12) capital letters (except K and F) denote time-mean quantities, lower-case letters (except μ_1 and g_i) designate fluctuating components and overbars indicate

Reynolds-averaged correlations. For constant quantities (μ_1 ρ_2 and g_i) the mean and instantaneous values are equal, whereas for all other variables the instantaneous value consists of a mean component and a fluctuating component (e.g., $Q + q$, $U + u$, $\rho_1 + \rho_1'$ ... etc.).

SECTION 3

TURBULENCE-KINETIC-ENERGY AND DISSIPATION-RATE EQUATIONS

The first step in the derivation of the equations of the fluid's turbulence kinetic energy ($k \equiv \overline{u_i u_i}/2$) and its dissipation rate ($\epsilon \equiv \nu_1 \overline{u_{i,k}\, u_{i,k}}$ where $\nu_1 = \mu_1/\rho_1$) is to obtain a transport equation for u_i by subtracting Eq. (6) from Eq. (1). The k equation is produced by multiplying the u_i equation throughout by u_i and then time-averaging. The ϵ equation is obtained by differentiating the u_i equation with respect to x_k, multiplying throughout by $2\nu_1 u_{i,k}$ and finally time-averaging.

The resulting k and ϵ equations are given in Appendix A. The closure of these equations is discussed in Sections 5 and 6, respectively.

SECTION 4

CLOSURE OF THE MOMENTUM EQUATIONS

The turbulent correlations appearing in Eqs. (6) and (7) are of five types:

1. Correlation of velocity fluctuations with those of the volume fraction or apparent density, e.g., $\overline{\phi_1 u_i}$ or $\overline{q_1 u_i}$;

2. The pressure interaction correlation $\overline{\phi_2 p_{,i}}$;

3. Multiple correlations among various components of velocity fluctuations with those of the apparent density, e.g., $\overline{q_1 u_i u_j}$;

4. Correlations of strain rate fluctuations with those of the volume fraction, e.g., $\overline{\phi_1 u_{i,j}}$;

5. Multiple correlations of various components of velocity fluctuations, e.g., $\overline{u_i u_j}$.

The first four types occur only due to the presence of the second phase; their modeling is discussed below.

According to Ref. 3 (Eq. (10)) and Ref. 12, we model the turbulent flux $\overline{\phi_1 u_i}$ by a gradient transport term and a convective transport term such that

$$\overline{\phi_1 u_i} = -(\nu_t/\sigma_\phi)\Phi_{1,i} - \tfrac{1}{2}\,\Phi_1(\nu_t/\sigma_\phi)_{,i} \quad , \qquad (13)$$

where ν_t is the kinematic eddy viscosity ($= c_\mu k^2/\epsilon$ with $c_\mu = 0.09$, Ref. 13, under the provisional assumption that it has the same value as in the case of constant density) and σ_ϕ is the turbulent Schmidt number of ϕ. Similarly we model $\overline{q_i u_i}$ by

$$\overline{q_1 u_i} = -(\nu_t/\sigma_q) Q_{1,i} - \frac{1}{2} Q_1 (\nu_t/\sigma_q)_{,i} \tag{14}$$

where σ_q is the turbulent Schmidt number of q. As long as experimental data for σ_q are not available we make a provisional assumption that σ_q is equal to σ_ϕ (= 1.0 for the sample calculation in Ref. 3).

According to Ref. 3 (Eqs. (11), (12), (14), (15) and (16)) and Refs. 14, 15 and 16 we model $\overline{\phi_2 p_{,i}}$ by

$$\begin{aligned}\overline{\phi_2 p_{,i}} = &- \rho_1 \frac{\varepsilon}{k}\left[c_{\phi 1} \overline{u_i \phi_1} - c_{\phi 2}\left(\frac{\overline{u_i u_\ell}}{k} - \frac{2}{3}\delta_{i\ell}\right)\overline{u_\ell \phi_1}\right] \\ &+ \rho_1 (0.8\, \overline{u_\ell \phi_1}\, U_{i,\ell} - 0.2\, \overline{u_\ell \phi_1}\, U_{\ell,i}) \\ &- \Big[-c_{\phi 3} \rho_1 k^{1/2}\, \overline{u_m \phi_1}\, e_m \\ &+ c_{\phi 4} \rho_1\, k^{3/2}/\epsilon\, U_{n,m}\, \overline{u_m \phi_1}\, e_n\Big]_{,i}\ ,\end{aligned} \tag{15}$$

where e_m and e_n are unit vectors and the approximate values of the coefficients are

$$c_{\phi 1} = 4.3,\quad c_{\phi 2} = 3.2,\quad c_{\phi 3} = 1.0,\quad c_{\phi 4} = 1.0\ . \tag{16}$$

According to Ref. 16 (Eq. (6.45)) we model $\overline{q_1 u_i u_j}$ by

$$\overline{q_1 u_i u_j} = -c_{\phi 5}(k/\epsilon)\left[\overline{u_i u_\ell}\,(\overline{u_j q_1})_{,\ell} + \overline{u_j u_\ell}\,(\overline{u_i q_1})_{,\ell}\right] \quad , \tag{17}$$

where the proportionality constant $c_{\phi 5}$ is approximately equal to 0.1.

The strain-rate volume-fraction correlations of the type $\overline{\phi_1 u_{i,j}}$ only appear multiplied by the molecular viscosity of the fluid and therefore will be neglected due to its relatively small magnitude.

The last correlation to be modeled in the momentum equations is that of the form $\overline{u_i u_j}$. Again, to be consistent with the present level of closure, this quantity will be calculated from (Eq. (18) in Ref. 3)

$$\overline{u_i u_j} = -\nu_t(U_{i,j}+U_{j,i}) + \frac{1}{3}\delta_{ij}\overline{u_m u_m} - \frac{2}{3}\nu_t\,\delta_{ij}U_{j,j} \ . \tag{18}$$

This completes the modeling of the momentum equations.

SECTION 5

CLOSURE OF THE TURBULENCE KINETIC-ENERGY EQUATION

The exact equation of the turbulence energy k for the carrier fluid appears in Appendix A and consists of 34 terms. They are classified into groups enclosed by square or curly brackets; each group is labeled according to its particular contribution to the conservation of k.

The various correlations in these groups range from second to fourth order. We decide at the outset on neglecting all fourth-order correlations such as $\overline{q_1 u_i (u_i u_j)_{,j}}$ and $\overline{u_i u_i u_j q_{1,j}}$. Also, the contribution to the diffusion of turbulence energy due to the pressure interaction $(\overline{u_i p})_{,i}$ will be neglected as it is of relatively small magnitude (Ref. 17). Now the remaining terms will be modeled.

The five transient terms will be collectively approximated by $(Q_1 k)_{,t}$. The convection terms require no approximation. The production group contains the correlations $\overline{q_1 u_i u_j}$ and $\overline{u_i u_j}$ which have been evaluated earlier by Eqs. (17) and (18). The pressure velocity-divergence correlation $\overline{pu_{i,i}}$ in the turbulent-diffusion group cannot be neglected here since $u_{i,i}$ does not vanish in two-phase flows. $\overline{pu_{i,i}}$ is evaluated following the approach of Ref. 18, thus (Eqs. (19) and (20) of Ref. 3))

$$- \overline{pu_{i,i}} = c_1 \frac{\rho_1}{2} (\frac{\varepsilon}{k}) (\overline{u_i u_i} - \frac{2k}{3}) + \frac{(c_2+8)\rho_1}{22} (P_{ii} - \frac{2P}{3})$$

$$+ \frac{(15c_1-1)\rho_1}{55} (2k\ U_{i,i}) - \frac{(4c_2-1)\rho_1}{11} (D_{ii} - \frac{2P}{3}) \quad (19)$$

where

$$\left.\begin{aligned} P_{ii} &= -2(\overline{u_i u_k}\, U_{i,k}) \quad , \\ P &= \frac{1}{2} P_{ii} \quad , \\ D_{ii} &= -2(\overline{u_i u_k}\, U_{k,i}) \quad , \\ c_1 &= 1.5 \ , \ c_2 = 0.4 \quad . \end{aligned}\right\} \tag{20}$$

The last two terms in the turbulent-diffusion group can be modeled as (Eqs. (21) to (25) and Eq. (10) of Ref. 3)

$$- \overline{u_i u_i (Q_1 u_j)_{,j}} - Q_1 \overline{u_i u_j u_{i,j}} = \Big\{ Q_1 \Big[(\nu_t/\sigma_k) k_{,j} + \frac{1}{2} k (\nu_t/\sigma_k)_{,j} \Big] \Big\}_{,j} \tag{21}$$

where the turbulent Schmidt number of k is taken as $\sigma_k = 1.0$ (Ref. 13, under the provisional assumption that it has the same value as in the case of constant density).

There are eight terms in the extra production and transfer group, the last two of which are neglected for being of fourth order. The remaining six terms are modeled next.

The second term, $-Q_1 U_i \, \overline{u_i u_{j,j}}$, is modeled following the proposal of Ref. 19 as $Q_1 U_i \nu_t U_{j,ij}$. The correlation of the form $\overline{u_i (q_1 u_j)_{,j}}$ which appears in the third and fourth terms is expanded as

$$\overline{u_i (q_1 u_j)_{,j}} = (\overline{u_i q_1 u_j})_{,j} - \overline{q_1 u_j u_{i,j}} \tag{22}$$

where the first term on the right is evaluated using Eq. (17), and the second term is modeled as

$$\overline{q_1 u_j u_{i,j}} = - c_{\phi 5} \frac{\varepsilon}{k} \left[\frac{\nu_t}{\sigma_q} Q_{1,i} + \frac{1}{2} Q_1 \left(\frac{\nu_t}{\sigma_q} \right)_{,i} \right] \tag{23}$$

where the provisional assumption on σ_q has already been mentioned in Section 4. The correlations $\overline{\phi_1 u_i}$ (in the first term) and $\overline{q_1 u_i}$ (in the fifth term) have been discussed earlier (Eqs. (13) and (14)). The sixth term is approximated by

$$U_i U_j \overline{u_i q_{1,j}} \cong U_i U_j \left[(\overline{u_i q_1})_{,j} - \overline{q_1 u_{i,j}} \right] \tag{24}$$

where $\overline{q_1 u_{i,j}}$ is neglected for being relatively smaller than $(\overline{u_i q_1})_{,j}$.

The extra dissipation group contains three terms which exist only due to the slip between the two phases.. According to Ref. 3 (Eq. (39)) and Refs. 20 and 21 we model the correlation $\overline{u_i (v_i - u_i)}$ by

$$\overline{u_i (v_i - u_i)} = -\frac{1}{2} \overline{u_i^2} \left(1 - \int_0^\infty \frac{\Omega_1 - \Omega_R}{\Omega_2} f(\omega) d\omega \right) \tag{25}$$

where ω is the frequency of turbulence and

$$\bar{u}_i^2 = \frac{1}{3} \overline{u_i u_i} = \frac{2}{3} k \tag{26}$$

$$\Omega_1 = (\omega/\alpha)^2 + \sqrt{6} (\omega/\alpha)^{3/2} + 3(\omega/\alpha) - \sqrt{6} (\omega/\alpha)^{1/2} + 1 \tag{27}$$

$$\Omega_2 = \beta^{-2}(\omega/\alpha)^2 + \sqrt{6}\,\beta^{-1}(\omega/\alpha)^{3/2} + 3(\omega/\alpha)$$

$$+ \sqrt{6}\,(\omega/\alpha)^{1/2} + 1 \tag{28}$$

$$\Omega_R = \left[(1-\beta)\omega/\alpha\beta\right]^2 \tag{29}$$

$$f(\omega) = 4(\pi)^{3/2}\,\lambda\left[\left(\frac{2}{3}k\right)^{-1/2}\right] e^{-(\lambda\omega)^2/(\frac{8}{3}k)} \tag{30}$$

in which

$$\alpha = 12\mu_1/\rho_1 d^2 \tag{31}$$

$$\beta = 3\rho_1/(2\rho_2 + \rho_1) \tag{32}$$

and λ is Taylor's microscale. The last term in the extra dissipation group contains the triple correlations $\overline{\phi_2 u_i u_i}$ and $\overline{\phi_2 u_i v_i}$ which can be modeled by Eqs. (17) and (42) of Ref. 3 as

$$\overline{\phi_2 u_i u_i} = -2c_{\phi 5}\,(k/\epsilon)\,\overline{u_i u_j}\,(\overline{u_i\phi_2})_{,j} \tag{33}$$

and

$$\overline{\phi_2 u_i v_i} = -c_{\phi 5}\,(k/\epsilon)\left[\overline{u_i v_j}(\overline{v_i\phi_2})_{,j} + \overline{v_i u_j}(\overline{u_i\phi_2})_{,j}\right] \tag{34}$$

where the double correlations on the right sides have been modeled earlier (Eqs. (13) and (18)).

The term $\mu_i\Phi_i\,\overline{u_i(u_{i,j}+u_{j,i})}_{,j}$ constitutes the dissipation of k due to viscous action, and if Φ_i is set to unity it is reduced

to the single-phase dissipation terms. This term is modeled as (Eq. (43) in Ref. 3)

$$\mu_1\Phi_1 \overline{u_i(u_{i,j}+u_{j,i})_{,j}} \cong -\rho_1\Phi_1\epsilon \qquad (35)$$

The six terms in the viscous diffusion and dissipation group will be neglected due to their relatively small magnitudes as compared to the terms in the turbulent diffusion group.

The first term in the field forces effects group is modeled by Eq. (14). Similarly the second term in the group can be modeled as

$$\overline{u_i f_{1i}} = -(\nu_t/\sigma_f)F_{1i,i} - \tfrac{1}{2}F_{1i}(\nu_t/\sigma_f)_{,i} \qquad (36)$$

where we make a provisional assumption that σ_f is equal to σ_ϕ.

SECTION 6

CLOSURE OF THE TURBULENCE-ENERGY DISSIPATION-RATE EQUATION

The exact equation of the dissipation rate of turbulence energy ϵ for the carrier fluid appears in Appendix A and consists of 52 terms. They are classified into groups similar to those of the k equation.

Again we neglect all fourth-order correlations as mentioned in the previous section.

All the terms in the first group, labeled Transient, are approximated collectively by $(Q_1\epsilon)_{,t}$.

The convection group consists of eight terms of which only the first and the second are of higher magnitude than the other six at large Reynolds number. This is based on an order of magnitude analysis (Ref. 22) which shows that the first and second terms are greater than the others by at least a factor of (ℓ/λ), which is of order $(R_\ell)^{1/2}$. Here ℓ is the length scale of the energy containing eddies, λ is a Taylor's microscale, and R_ℓ is the Reynolds number based on ℓ.

The third term in the production group is decomposed as

$$-2\nu_1 Q_1(\overline{u_i u_{j,k} u_{i,k}})_{,j} = -2\nu_1 Q_1(\overline{u_{i,j} u_{j,k} u_{i,k}} + \overline{u_i u_{j,k} u_{i,k}} + \overline{u_i u_{j,k} u_{i,kj}}) \qquad (37)$$

The third term on the right side of Eq. (37) and the second term in the production group differ only in their signs and thus cancel each other. The first term on the right side of

Eq. (37), which represents the production of ϵ by self-stretching of the vortex tubes, is the dominant one at large Reynolds number. It is larger than the second term by a factor of R_ℓ and larger than the first term in the production group by a factor of $R_\ell^{1/2}$.

We, therefore, retain only $-2\nu_1 Q_1 \overline{u_{i,j}u_{j,k}u_{i,k}}$ as the main generation of ϵ. This term and the extra production terms are modeled collectively as

$$-2\nu_1 Q_1 \overline{u_{i,j}u_{j,k}u_{i,k}} + \text{extra production terms} = c_{\epsilon 1} G_k \epsilon / k \ , \quad (38)$$

where G_k is the total production of k discussed in Section 5, and $c_{\epsilon 1}$ is a constant of value 1.43 (Ref. 13, under the provisional assumption that it has the same value as in the case of constant density). "Total" here means the production terms which are common to the single-phase and two-phase k equations in addition to the extra production and transfer terms.

The turbulent diffusion group contains six terms. At high Reynolds number only the last two terms will be retained; they are larger by at least a factor of $R_\ell^{1/2}$ than the other terms. These two terms will be modeled collectively as

$$-2\nu_1 (Q_1 \overline{u_j u_{i,k} u_{i,jk}} + Q_{1,j} \overline{u_j u_{i,k} u_{i,k}})$$

$$= \left[Q_1 (\nu_t/\sigma_\epsilon)\epsilon_{,j} + \tfrac{1}{2} Q_1 \epsilon (\nu_t/\sigma_\epsilon)_{,j}\right]_{,j} \quad (39)$$

All the terms in the extra production group except the mean pressure gradient term are smaller than the main production term, modeled in Eq. (38), by at least a factor of $R_\ell^{-1/2}$ and

thus can be neglected. The mean pressure gradient term is included in the production term (Eq. (38)).

The first term in the viscous diffusion and dissipation group represents the main dissipation of ϵ; it reduces to the single-phase form when Φ_1 equals unity. This term is larger than the other terms in the group by a factor of $R_\ell^{3/2}$ and thus it is the only one retained. Now the total dissipation of ϵ includes this term in addition to the extra dissipation terms. They are modeled collectively as $Q_1(\epsilon/k)(c_{\epsilon 2}\epsilon + c_{\epsilon 3}\epsilon_e)$ where ϵ_e represents the extra dissipation terms appearing in the k equation, $c_{\epsilon 2}$ is a constant of a value about 1.92 (Ref. 13) and $c_{\epsilon 3}$ is a constant of value 1.2 (Ref. 3) (both under the provisional assumption that they have the same values as in the case of constant density).

Both terms in the field forces effects group appear multiplied by the kinematic molecular viscosity of the fluid and therefore will be neglected due to their relatively small magnitudes.

SECTION 7

A SAMPLE APPLICATION

As an example of the application of the modeled k and ϵ equations, let us consider the motion of the dusty air during a nuclear explosion (Ref. 6). In order to understand the essential features of the complicated phenomena, some highly idealized and well-controlled laboratory tests have been performed or planned such as high speed wind above a sand bed in a wind tunnel (Ref. 23) and shock wave sweeping a sand bed in a shock tube (Ref. 24). To interpret and correlate the results from such tests, we may use the two-dimensional version of the present model in Cartesian coordinates x and y. The final form of the modeled k and ϵ equations is given in Appendix B (under the assumptions that the diffusional fluxes in y direction are much larger than those in x direction, that the effective coefficient $K = 1$ and that $g_y = g$ and $g_x = F_{1x} = F_{1y} = 0$). The remaining modeled equations contain two mean continuity equations (from Eqs. (8) and (9)), one mean global continuity equation (Eq. (11)), four mean momentum equations (from Eqs. (6) and (7) and Section 4) and one mean equation of state such as the perfect gas equation

$$P = R\rho_1 T_1 \tag{40}$$

where R is the specific gas constant and T_1 is the absolute temperature of the air. Thus, for isothermal problems (T_1 = constant) we have ten equations for ten unknowns Q_1, Q_2, ρ_1, U_x, U_y, V_x, V_y, P, k and ϵ (from which $\Phi_1 = Q_1/\rho_1$ and $\Phi_2 = Q_2/\rho_2$ can be readily obtained). With proper initial and boundary conditions, these equations can be solved numerically by a marching finite-difference procedure described in Ref. 5.

The results of such calculation will be presented in a forthcoming report.

If T_1 is variable and the energy exchange between the air and dust can be neglected we need to include a mean energy equation for the air in the numerical solution procedure.

If the energy exchange between the air and dust (at temperature T_2) is not negligible, we need to include the mean energy equations for both air and dust in the numerical solution.

SECTION 8

CONCLUSION

The k - ϵ turbulence model for an incompressible dilute suspension of Ref. 3 has been extended to a compressible dispersed two-phase flow by introducing the apparent densities Q_1 and Q_2 and the material density ρ_1 as new variables. The fluid has variable density and the particles have constant density. This allows the application of the model to a wider class of practical problems.

As in Ref. 3, the k and ϵ equations are first rigorously derived from the two-phase momentum equations and then their closure is provided. This is in contrast to the usual approach based on ad hoc modifications of the single-phase turbulence-kinetic-energy and length-scale equations.

The proposed closure of the equations accounts for the interaction between the two phases and its influence on the turbulence structure. Sparseness of experimental data for variable-density flows necessitates some provisional assumptions that forms and values of the coefficients in the turbulent correlations of constant-density flows apply to those of variable-density flows. Such assumptions indicate areas of needed experimental investigations which, when completed, can in turn modify the present work and enhance its validity.

SECTION 9

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APPENDIX A

EXACT EQUATIONS OF KINETIC ENERGY OF TURBULENCE AND DISSIPATION RATE OF THAT ENERGY

The exact equation of the turbulence kinetic energy ($k \equiv \overline{u_i u_i}/2$) of the carrier fluid is

$$\underbrace{\Big[(Q_1 \overline{u_i u_i}/2)_{,t} + Q_{1,t}\overline{u_i u_i}/2 + U_{i,t}\overline{u_i q_1} + U_i \overline{u_i q_{1,t}}}_{\text{Transient}}$$

$$+ \overline{u_i (q_1 u_i)_{,t}}\Big] + \underbrace{\Big[(Q_1 U_j k)_{,j} + k(Q_1 U_j)_{,j}\Big]}_{\text{Convection}} =$$

$$- \underbrace{\Big[(Q_1 U_i)_{,j}\overline{u_i u_j} + U_{i,j}\overline{q_1 u_i u_j} + U_{j,j}\overline{q_1 u_i u_i}\Big]}_{\text{Production}}$$

$$- \underbrace{\Big\{(1-K\Phi_2)\Big[\overline{(u_i p)}_{,i} - \overline{p u_{i,i}}\Big] + K\overline{\phi_1 u_i P_{,i}} + \overline{u_i u_i (Q_1 u_j)_{,j}}}_{\text{Turbulent Diffusion}}$$

$$+ Q_1 \overline{u_i u_j u_{i,j}}\Big\} - \Big[KP_{,i}\overline{\phi_1 u_i} + Q_1 U_i \overline{u_i u_{j,j}} + U_i \overline{u_i (q_1 u_j)_{,j}}$$

$$\underbrace{+ U_j \overline{u_i (q_1 u_i)_{,j}} + (U_i U_j)_{,j}\overline{q_1 u_i} + U_i U_j \overline{u_i q_{1,j}}}_{\text{Extra Production and Transfer}}$$

$$+ \overline{q_1 u_i (u_i u_j)_{,j}} + \overline{u_i u_i u_j q_{1,j}}\Big] + KF\Big[\Phi_2 \overline{u_i (v_i - u_i)}$$

$$\underbrace{+ (V_i - U_i)\overline{\phi_2 u_i} + \overline{\phi_2 u_i (v_i - u_i)}\Big]}_{\text{Extra Dissipation}} + \underbrace{\mu_1\Big[\Phi_1 \overline{u_i (u_{i,j} + u_{j,i})_{,j}}\Big]}_{\text{Dissipation}}$$

$$+ \mu_1 \Big\{ \Phi_{1,j} \overline{u_i(u_{i,j}+u_{j,i})} + \overline{\left[(U_{i,j}+U_{j,i})\phi_1\right]_{,j} u_i}$$

$$+ \overline{\left[(u_{i,j}+u_{j,i})\phi_1\right]_{,j} u_i} - \tfrac{2}{3}\Big[\overline{(\Phi_1 u_{\ell,\ell})_{,i} u_i} + \overline{(\phi_1 U_{\ell,\ell})_{,i} u_i}$$

Viscous Diffusion and Dissipation

$$+ \overline{(\phi_1 u_{\ell,\ell})_{,i} u_i}\Big]\Big\} + \Big[g_i \overline{u_i q_1} + \overline{u_i f_{1i}}\Big] \ .$$

Field Forces Effects

The exact equation of the dissipation rate of k ($\epsilon \equiv \nu_1 \overline{u_{i,k} u_{i,k}}$) is

$$2\nu_1 \Big[\overline{(Q_1 u_i)_{,kt} u_{i,k}} + \overline{(q_1 U_i)_{,kt} u_{i,k}} + \overline{(q_1 u_i)_{,kt} u_{i,k}}\Big]$$

Transient

$$+ \Big\{(Q_1 U_j \epsilon)_{,j} + \epsilon (Q_1 U_j)_{,j} + 2\nu_1 \Big[(Q_1 U_i)_{,j} \overline{u_{i,k} u_{j,k}}$$

$$+ (Q_1 U_i)_{,k} \overline{u_{j,j} u_{i,k}} + (Q_1 U_i)_{,jk} \overline{u_{i,k} u_j} + (Q_1 U_j)_{,k} \overline{u_{i,k} u_{i,j}}$$

Convection

$$+ (Q_1 U_j)_{,jk} \overline{u_i u_{i,k}} + Q_1 U_i \overline{u_{i,k} u_{j,jk}}\Big]\Big\} = -2\nu_1 \Big[Q_1 \overline{u_{i,k} u_{i,k} u_{i,j}}$$

$$- Q_1 \overline{u_i u_{i,jk} u_{j,k}} + Q_1 (\overline{u_i u_{j,k} u_{i,k}})_{,j}\Big]$$

Production

$$+ 2\nu_1 KF \Big\{ \overline{\left[\Phi_2 (v_i - u_i)\right]_{,k} u_{i,k}} + \overline{\left[\phi_2 (V_i - U_i)\right]_{,k} u_{i,k}}$$

Extra Dissipation

$$+ \overline{\left[\phi_2 (v_i - u_i)\right]_{,k} u_{i,k}} \Big\} - 2\nu_1 \Big[Q_{1,jk} \overline{u_i u_j u_{i,k}}$$

$$+ Q_{1,k}\overline{(u_i u_{j,j} + u_j u_{i,j})u_{i,k}} + Q_{1,j}\overline{u_i u_{j,k} u_{i,k}} + \overline{p_{,ki} u_{i,k}}$$

Turbulent Diffusion

$$+ Q_1\overline{u_j u_{i,k} u_{i,jk}} + Q_{1,j}\overline{u_j u_{i,k} u_{i,k}}\Big] + 2\nu_1\Big\{\overline{(K\Phi_2 p_{,i})_{,k} u_{i,k}}$$

$$- KP_{,ki}\overline{\phi_1 u_{i,k}} - P_{,i}\overline{u_{i,k}(K\phi_1)_{,k}} - \overline{(K\phi_1 p_{,i})_{,k} u_{i,k}}$$

$$- \overline{u_{i,k}\left[(q_1 U_i)_{,j} u_{j,k} + (q_1 U_i)_{,k} u_{j,j}\right]} - U_i\overline{q_1 u_{i,k} u_{j,jk}}$$

$$- \overline{u_j u_{i,k}(q_1 U_i)_{,jk}} - U_j\overline{q_1 u_{i,k} u_{i,jk}} - \overline{u_i (q_1 U_j)_{,jk} u_{i,k}}$$

Extra -

$$- \left[\overline{(q_1 U_j)_{,j} u_{i,k} u_{i,k}} + \overline{(q_1 U_j)_{,k} u_{i,j} u_{i,k}}\right]$$

- Production

$$- \overline{\left[U_i U_j q_{1,jk} + q_{1,k}(U_i U_{j,j} + U_j U_{i,j})\right] u_{i,k}}$$

$$- \left[q_1 (U_{i,j} U_j)_{,k} + U_i U_{j,k} q_{1,j} + q_1 (U_{j,j} U_i)_{,k}\right]\overline{u_{i,k}}$$

$$- U_j U_{i,k}\overline{q_{1,j} u_{i,k}} - \overline{\left[u_i u_j q_{1,jk} + q_{1,k}(u_i u_{j,j} + u_j u_{i,j})\right] u_{i,k}}$$

$$- \overline{\left[q_1 (u_i u_{j,k})_{,j} + u_i u_{j,k} q_{1,j} + q_1 (u_j u_{i,k})_{,j} + u_j u_{i,k} q_{1,j}\right] u_{i,k}}\Big\}$$

$$+ 2\rho_1 \nu_1^2 \Big\{\overline{\left[\phi_1 (u_{i,j} + u_{j,i})\right]_{,jk} u_{i,k}} + \overline{\left[\phi_1 (U_{i,j} + U_{j,i})\right]_{,jk} u_{i,k}}$$

$$+ \overline{\left[\phi_1 (u_{i,j} + u_{j,i})\right]_{,jk} u_{i,k}} - \frac{2}{3}\Big[\overline{(\phi_1 u_{\ell,\ell})_{,ik} u_{i,k}}$$

Viscous Diffusion and Dissipation

$$+ \overline{(\phi_1 U_{\ell,\ell})_{,ik} u_{i,k}} + \overline{(\phi_1 u_{\ell,\ell})_{,ik} u_{i,k}}\Big]\Big\}$$

$$+ 2\nu_1\left[g_i\overline{q_{1,k} u_{i,k}} + \overline{f_{1i,k} u_{i,k}}\right] .$$

Field Forces Effects

APPENDIX B
THE MODELED FORM OF κ AND ε EQUATIONS

The modeled conservation equations of the kinetic energy and the dissipation rate of that energy for the carrier fluid in the sample application are listed here.

(i) The k equation:

$$\left|Q_1 k_{,t}\right| + Q_1 \left|U_x k_{,x} + U_y k_{,y}\right|$$

Transient Convection

$$= \left\{\left[\nu_t U_{x,y}(Q_1 U_x)_{,y} - \frac{2}{3} k (Q_1 U_y)_{,y}\right] + c_{\phi 5}\left(\frac{k}{\varepsilon}\right)\left(\frac{\nu_t}{\sigma_\phi} Q_{1,y}\right)_{,y} \left(\nu_t U^2_{x,y} - \frac{4}{3} k U_{y,y}\right)\right\}$$

Production (P)

$$- \left\{Q_1\left[\frac{6.4}{11}\left(\nu_t U^2_{x,y} - \frac{2}{3} k\, U_{y,y}\right) - \frac{43}{55} k\, U_{y,y}\right] + \left[Q_1\left(\frac{\nu_t}{\sigma_k}\right)k_{,y} + \frac{1}{2} Q_1 k\left(\frac{\nu_t}{\sigma_k}\right)_{,y}\right]_{,y}\right\}$$

Diffusion

$$+ \left[-\left(\frac{P_{,y}}{\rho_2}\right)\left(\frac{\nu_t}{\sigma_\phi}\right)Q_{2,y} + \nu_t Q_1 U_y U_{y,yy} + c_{\phi 5} U_x \left[\left(\frac{k}{\varepsilon}\right)\nu_t U_{x,y}\left(\frac{\nu_t}{\sigma_\phi} Q_{1,y}\right)_{,y}\right]_{,y}\right.$$

$$- 2c_{\phi 5} U_y \left\{\left[\frac{4}{3}\left(\frac{k^2}{\varepsilon}\right)\left(\frac{\nu_t}{\sigma_\phi} Q_{1,y}\right)_{,y}\right]_{,y} + \left(\frac{\varepsilon}{k}\right)\left[\left(\frac{\nu_t}{\sigma_\phi} Q_{1,y}\right) + 0.5\, Q_1\left(\frac{\nu_t}{\sigma_\phi}\right)_{,y}\right]\right.$$

Extra Production (P_e)

$$\left.\left. + \left(U_y^2\right)_{,y}\left(\frac{\nu_t}{\sigma_\phi}\right)Q_{1,y} + U_y^2\left[\left(\frac{\nu_t}{\sigma_\phi}\right)Q_{1,y}\right]_{,y}\right\}\right]$$

$$- \left(\frac{F}{\rho_2}\right)\left[kQ_2\left(1 - \int_0^\infty \frac{\Omega_1-\Omega_R}{\Omega_2}\, f(\omega)\,d\omega\right) - \left(V_y - U_y\right)\left(\frac{\nu_t}{\sigma_\phi}\right)Q_{2,y}\right.$$

Extra Dissipation (ε_e)

$$\left. + c_{\phi 5}\left(\frac{F}{3\rho_2}\right)\left(\frac{k^2}{\varepsilon}\right)\left(1 - \int_0^\infty \frac{\Omega_1-\Omega_R}{\Omega_2}\, f(\omega)\,d\omega\right)\left(\frac{\nu_t}{\sigma_\phi}\, Q_{2,y}\right)_{,y}\right]$$

$$- \left[Q_1 \varepsilon\right] - g\left[\left(\frac{\nu_t}{\sigma_\phi}\right)Q_{1,y} + \frac{1}{2}\, Q_1\left(\frac{\nu_t}{\sigma_\phi}\right)_{,y}\right] .$$

Dissipation Field Forces Effects

(ii) The ε equation:

$$\left[Q_1 \varepsilon_{,t}\right] + Q_1\left[U_x \varepsilon_{,x} + U_y \varepsilon_{,y}\right]$$

Transient Convection

$$= \left[c_{\varepsilon 1}(\varepsilon/k)\left(P+P_e\right)\right] + \left[Q_1\left(\nu_t/\sigma_\varepsilon\right)\varepsilon_{,y} + \frac{1}{2}\, Q_1\, \varepsilon\left(\nu_t/\sigma_\varepsilon\right)_{,y}\right]_{,y}$$

Total Production Diffusion

$$- \left[Q_1(\varepsilon/k)\left(c_{\varepsilon 2}\, \varepsilon + c_{\varepsilon 3}\, \varepsilon_e\right)\right] .$$

Total Dissipation

The notations used in the partial derivatives have been explained in Section 2 of the text, thus $(\ )_{,t}$ means $\partial(\ )/\partial t$, $(\ )_{,x}$ means $\partial(\ )/\partial x$ and $(\ )_{,y}$ means $\partial(\ )/\partial y$, where x and y are the distances along the horizontal and vertical directions, respectively. The values of the constants appearing in the two equations are:

$c_{\varepsilon 1} = 1.43$, $c_{\varepsilon 2} = 1.92$, $c_{\varepsilon 3} = 1.2$, $c_{\phi 5} = 0.1$, $\sigma_k = 1.0$,

$\sigma_\varepsilon = 1.3$, $\sigma_\phi = 1.0$.

APPENDIX C

NOMENCLATURE

c_1, c_2, c_μ, $c_{\epsilon 1}$, $c_{\epsilon 2}$, $c_{\epsilon 3}$, $c_{\phi 1}$, $c_{\phi 2}$, $c_{\phi 3}$, $c_{\phi 4}$, $c_{\phi 5}$

constants in the turbulence model

d	particle diameter
D_{ii}	an expression defined in Eqs. (20)
$\underline{e}_m$, $\underline{e}_n$	unit vectors
$f(\omega)$	an expression defined in Eq. (30)
f_i	fluctuating component of body forces other than that due to gravity
F_i	instantaneous (in Eqs. (1) and (2)) or time-mean component of body forces other than that due to gravity
F	interface friction coefficient
g_i	component of gravitational acceleration
G_k	total production of k
k	turbulence kinetic energy of the fluid
K	local effectiveness of momentum transfer from the particle phase to the fluid
ℓ	length scale
p	pressure fluctuation
P	instantaneous (in Eqs. (1) and (2)) or time-mean pressure
P	an expression defined in Eqs. (20)
P	production (in Appendix B)
P_e	extra production (in Appendix B)
q	fluctuation of the apparent density
Q	instantaneous (in Eqs. (1)-(5)) or time-mean apparent density
R	specific gas constant
R_ℓ	Reynolds number based on ℓ

t	time
T	absolute temperature
u_i	fluctuating velocity component of the fluid
U_i	instantaneous (in Eqs. (1)-(3)) or time-mean velocity component of the fluid
v_i	fluctuating velocity component of the particle phase
V_i	instantaneous (in Eqs. (1)-(4)) or time-mean velocity component of the particle phase
x_i	rectangular spatial coordinate
x	horizontal coordinate
y	vertical coordinate

Greek symbols

α	an expression defined in Eq. (31)
β	an expression defined in Eq. (32)
δ_{ij}	Kronecker symbol
ϵ	dissipation rate of k
ϵ_e	extra dissipation terms in the k equation
λ	Taylor's microscale
μ	viscosity
ν	kinematic viscosity
ν_t	kinematic eddy viscosity
ρ	material density
σ_ϕ, σ_q, σ_k, σ_f, σ_ϵ	turbulent Schmidt numbers
ϕ	fluctuation of the volume fraction
Φ	instantaneous (in Eqs. (1)-(5)) or time-mean volume fraction
ω	frequency of turbulence
Ω_1	an expression defined in Eq. (27)
Ω_2	an expression defined in Eq. (28)
Ω_R	an expression defined in Eq. (29)

Subscripts

1	fluid phase
2	particle phase
,t	partial derivative with respect to t
,j	partial derivative with respect to x_j

Superscript

———	time-averaged value
′	fluctuating component

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